

1. 1 punct din oficiu

a.  $m_g \leq m_a$ .....1p

$\sqrt{xy} < \frac{x+y}{2} \cdot (x+y)$ .....1p

Finalizare .....1p

b. Utilizând inegalitatea din punctul a)

$\frac{\sqrt{2}}{3} + \frac{\sqrt{6}}{5} + \frac{\sqrt{12}}{7} + \dots + \frac{\sqrt{1005 \cdot 1006}}{2011} \leq \frac{1}{2} + \frac{1}{2} + \dots + \frac{1}{2}$ .....2p

Finalizare .....1p

2. 1 punct din oficiu

a.  $10^{2010} - 10^{2009} = 10^{2009}(10 - 1) = 9 \cdot 10^{2009}$ .....1p

$90 \cdot 10^{2008} = (2^2 + 3^2 + 4^2 + 3^2 + 5^2 + 6^2)(10^{1004})^2$ .....1p

Finalizare.....1p

b)

$10^{2012} = 100 \cdot 10^{2010} = (1^3 + 2^3 + 3^3 + 4^3) \cdot (10^{670})^3$ .....2p

Finalizare.....1p

3. 1 punct din oficiu

$\triangle ABF \equiv \triangle CBE$  (LUL)  $\Rightarrow AF = CE, PE = CE$  (ip),  $PF = AF$  (ip)  $\Rightarrow PE = PF$ .....2p

$\triangle PEB \equiv \triangle PFB$  (LLL)  $\Rightarrow \sphericalangle EPB \equiv \sphericalangle FBP \Rightarrow (BP \text{ bisectoarea } \sphericalangle ABC)$  (1).....2p

ABCD romb  $\Rightarrow$  (BD bisectoarea  $\sphericalangle ABC$ ) .....1p

Finalizare .....1p

4. 1 punct din oficiu

Construim  $CN \parallel PQ \parallel BM, M, N \in AA'$  .....1p

$\triangle CA'N \equiv \triangle MA'B \Rightarrow A'N = A'M$  .....1p

$PQ \parallel BM \Rightarrow \frac{BP}{PA} = \frac{MD}{DA}$ , respectiv  $\frac{CQ}{QA} = \frac{ND}{DA}$  .....2p

$\frac{BP}{PA} + \frac{CQ}{QA} = \frac{MD}{DA} + \frac{ND}{DA} = \frac{MD + ND}{DA} = 2 \frac{DA'}{DA} = 2 \frac{AA' - DA}{DA} = 2 \left( \frac{AA'}{DA} - 1 \right) = 2 \left( \frac{y}{x} - 1 \right)$ .....2p