

BAREM - clasa a VI-a -

Problema 1.

$a = 0$ sau $b = 0$ sau $c = 0 \Rightarrow a = b = c = 0$	0.5p
Fie $a, b, c > 0, a \leq b \leq c$	1p
$abc = a + b + c \leq 3c \Rightarrow c(ab - 3) \leq 0$	1.5p
$0 < ab \leq 3 \Rightarrow ab \in \{1, 2, 3\}$	1.5p
$ab = 1 \Rightarrow a = b = 1 \Rightarrow 2 + c = c$ -imposibil	0.5p
$ab = 2 \Rightarrow a = 1, b = 2 \Rightarrow 3 + c = 2c \Rightarrow c = 3$	0.5p
$ab = 3 \Rightarrow a = 1, b = 3 \Rightarrow 4 + c = 3c \Rightarrow c = 2$ -imposibil ($b \leq c$)	0.5p
Soluțiile sunt: $(0, 0, 0), (1, 2, 3), (1, 3, 2), (2, 1, 3), (2, 3, 1), (3, 1, 2), (3, 2, 1)$	1p
Total	7p

Problema 2.

$A = \overline{abcd}, B = \overline{dbca}, a, d \neq 0$	0.5p
$A \neq B$	0.5p
Fie $A > B$.	
$A - B = 999(a - d) \Rightarrow a > d$	0.5p
$7 \mid a - d \Rightarrow a = 8, d = 1$ sau $a = 9, d = 2$	1.5p
1) $a = 8, d = 1$	
$63 \mid 10b + c \Rightarrow b = 0, c = 0$ sau $b = 6, c = 3$	1.5p
2) $a = 9, d = 2$	
$9 \mid 10b + c + 2$ și $7 \mid 10b + c \Rightarrow b = 0, c = 7$ sau $b = 7, c = 0$	1p
$b = 0, c = 7$ -nu convine	0.5p
Cazul $A < B$ -analog	0.5p
Soluțiile:	
$A = 8001, B = 1008$ (sau invers), $A = 8631, B = 1368$ (sau invers), $A = 9702, B = 2708$ (sau invers)	0.5p
Total	7p

Problema 3.

Fie $n \in N^*$.	
$\frac{x+y}{n} = \frac{x+z}{n+1} = \frac{y+z}{n+2} = k(1)$	0.5p
$k \in N^*$	1p
$2x = k(n - 1), 2y = k(n + 1), 2z = k(n + 3)$	1p
$k \mid 2x, 2y, 2z \Rightarrow k \mid 2$	1p
$k = 1$ -nu convine	0.5p
$k = 2 \Rightarrow x = n - 1, y = n + 1, z = n + 3$	0.5p
$n \geq 3$	0.5p
$n = 3$ -nu convine	0.5p
$n = 4 \Rightarrow x = 3, y = 5, z = 7$	0.5p
$n \geq 5$ -nu convine	1p
Total	7p

Problema 4.

Cazul I: $D \in (AM)$

a) $\triangle AMB \equiv \triangle AMC$ (LLL) 1p

$AM \perp BC$ 1p

$\triangle DMB \equiv \triangle DMC$ (CC) $\Rightarrow [DB] \equiv [DC]$ 1p

b) Fie $AE \perp BD, E \in BD$ și $AF \perp CD, F \in CD$

$\triangle DMN \equiv \triangle DMP$ (IC) 1p

(DM bisectoarea \widehat{BDC}) 0.5p

(DM bisectoarea \widehat{FDE}) 0.5p

$[AF] \equiv [AE]$ 1p

Cazul II: $M \in (AD)$

a) Analog cazul I, a) 0.5p

b) Analog ca la cazul I, b) (\widehat{BDC} și \widehat{FDE} coincid) 0.5p

Total **7p**