Bistrita- Etapa județeană- 12.03.2011 Clasa a VI-a

Problema 1

$8+8\cdot 9+8\cdot 9^2+...+8\cdot 9^{2010}=8(\underbrace{1+9+9^2+...+9^{2010}}_{S})$ 1p

$-\begin{cases} S=1+9+9^2+...+9^{2010} \;/\cdot 9 \\ 9S=9+9^2+...+9^{2010}+9^{2011} \end{cases}$

$8S=9^{2011}-1$ 1p

$8S=x^{2011}-1$

$9^{2011}-1=x^{2011}-1 \Rightarrow x=9$ 1p

$A=9+99+999+...+\underbrace{\overline{999...9}}_{2011\,cifre}$

$A=(10-1)+(100-1)+(1000-1)+...+(\underbrace{\overline{100000...0}}_{2011\,cifre\,de\,0}-1)$ 1p

$A=10+100+...+\underbrace{\overline{100000...0}}_{2011\,cifre\,de\,0}-(\underbrace{1+1+...+1}_{2011})$

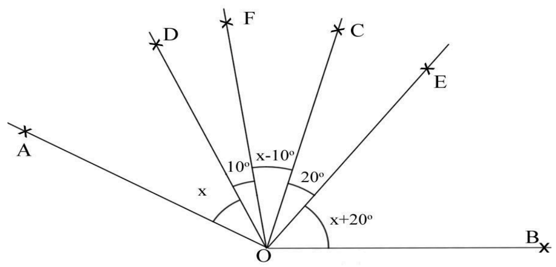
$A=11...10-2011$ 1p

$A=11...109099$ 1p

$Suma\,cifrelor=2007+27=2034$ 1p

$$\begin{array}{r} 111...111110\ - \\ 2011 \\ \hline \underbrace{111...1}_{2007}09099 \end{array}$$

Problema 2



$m(\angle AOB)\overset{not}{=}x$ 1p

$\left.\begin{array}{l} m(\angle AOF)=x+10^o \\ m(\angle AOD)=m(\angle DOC) \\ m(\angle FOC)=x-10^o \end{array}\right\}$ 2p

$m(\angle AOF)=m(\angle FOE) \Leftrightarrow x+10^o=x-10^o+m(\angle EOC) \Rightarrow$

$\Rightarrow m(\angle EOC)=20^o$ 1p

$m(\angle BOE)=m(\angle EOD) \Leftrightarrow m(\angle BOE)=10^o+x-10^o+20^o$ 1p

$\left.\begin{array}{l} m(\angle AOD)+m(\angle DOF)+m(\angle FOC)=m(\angle COE)+m(\angle BOE) \\ x+10^o+x-10^o=20^o+x+20^o \\ 2x=40^o+x \\ x=40^o \end{array}\right\}$ 1p

$Deci\ \ m(\angle AOB)=(40^o+20^o+20^o)\cdot 2=160^o$ 1p

Problema 3

$\left.\begin{array}{l} \frac{1}{2}<\frac{2}{3} \\ \frac{3}{4}<\frac{4}{5} \\ .... \\ \frac{99}{100}<\frac{100}{101} \end{array}\right\}$ 2p

$P<P_1 \;/\cdot P \Rightarrow P^2<P\cdot P_1$ 1p

$P\cdot P_1=\frac{1}{2}\cdot\frac{3}{4}\cdot\frac{5}{6}\cdot...\cdot\frac{99}{100}\cdot\frac{2}{3}\cdot\frac{4}{5}\cdot...\cdot\frac{98}{99}\cdot\frac{100}{101}=\frac{1}{101}$ 1p

$P\cdot P_1=\frac{1}{101}<\frac{1}{100}$

$\left.\begin{array}{l} P^2<P\cdot P_1 \\ P\cdot P_1<\frac{1}{100} \end{array}\right| \Rightarrow P^2<\frac{1}{100}$ 2p

$Deci\ \ P<\frac{1}{10}$ 1p